**Terminal Velocity Spreadsheet Modeling Supplemental Activities:**

**Objectives:** To write a program that will model motion through air and predict terminal velocity for objects. This is predicted given: cross-sectional area, mass, and drag coefficient.

1. To supplement the spreadsheet program to track position.
2. Simulate direction change in 1D gravitationally accelerated motion with drag force.
3. Predict the theoretical fall time through air from a given height.

**Discussion:** One assumption students are often taught is to disregard air resistance in any problem involving freefall. For massive objects at low speeds, this assumption is good. However, in reality air resistance begins affecting the motion of objects at relatively low speeds and must be considered to accurately model the motion of an object in freefall. When an object encounters air resistance as it falls, its acceleration begins to drop below 9.8 m/sec2.

As a falling object’s velocity increases the drag force it experiences increases and approaches the weight of the object. When these two forces become balanced (drag force = weight) and the object stops accelerating. When this happens, the object has reached its **terminal velocity**.

Since drag force (Fd) is proportional to the cross-sectional area (CSA) of an object and proportional to the velocity squared (v2), both must be considered in calculating it. To solve this analytically would require calculus (differential equations) which can be avoided using a spreadsheet that breaks up the motion into many small intervals which can be assumed to have uniform acceleration making the following equations valid.

**Governing Equations:**

** Eq. 1.1**

** Eq. 1.2**

** Eq. 1.3**

 ** Eq. 1.4**

$∆P=\frac{\left(V\_{i}+V\_{f}\right)}{2}\*t$ **Eq. 1.5**

$ P\_{f}=P\_{i}+∆P$ **Eq. 1.6**

Where **FD** is drag force (N); **c** is the drag coefficient (kg/m3); **A** is object cross-sectional area (m2); **v** is velocity (m/s); **w** is object weight (N); **m** is object mass (kg); **g** is 9.8 m/sec2; **a** is acceleration (m/s2); **Fnet** is net force (N); **t** is time increment (sec); Pi is initial position (m), Pf is final position (m), $∆P$is displacement (m)

**Setting up the spreadsheet:** Open Microsoft EXCEL.

1. Set up your spreadsheet **exactly** as it appears below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| 1 | Mass (kg) | CSA (m2) | Drag Coefficient (kg/m3) | Time Increment (sec) |  |  |
| 2 |   |   |   |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 | Time (sec) | Velocity (m/sec) | Weight (N) | Drag Force (N) | Net Force (N) | Acceleration (m/sec^2) |

1. The time increment will be given. It varies with the object being viewed. Objects that reach their terminal velocity quickly like a piece of paper or feather will require shorter time increments. To start use a time increment of 0.5 seconds which can be entered into cell D2 (do not include units). In cell A5, type “0”. In cell A6 type “ =$D$2+A5” **(Do not include the quotation marks**) Select cell A6 and then grab the fill handle in the lower right hand corner of the cell and drag down to cell 95. Column A should now count time by increments specified in D2.
2. In cell B5 type “0” for the initial velocity. **(Do not include the quotation marks.)** In cell B6 type “=B5+F5\*$D$2” to calculate the new velocity from Eq. 1.4.
3. Using Eq. 1.2 in cell C5 type “=9.8\*$A$2” for the weight of the object.
4. In cell D5 type “=$C$2\*$B$2\*B5^2”. This is Eq. 1.1 the equation for drag force (area times velocity squared times drag coefficient.)
5. In cell E5 type “=C5-D5” to sum the forces (weight and drag force).
6. In cell F5 type “= E5/$A$2” to calculate acceleration of the object using Newton’s 2nd Law (Eq. 1.3). Entering a mass will prevent a “divide by zero” error. Enter “95” in cell A2 for the mass of the first object.
7. Now your spreadsheet is almost complete. Go to the fill handle in the lower right hand corner of the bottom cell in each column and double click each fill handle (starting from the left) so the columns fill in to cell 95. ***Be sure to click the lower formula containing cell in any given column.***
8. Create a graph of the velocity (Y) vs. time (X) for the object in question.

 a. Click on the Insert tab at the top of the screen.

 b. Select the Scatter icon (looks like dots on a coordinate axis) and choose the graph type without the data points on it

c. To select the data **right click** the chart area and choose “Select Data” from the drop down menu. Remove any data series that is automatically selected. Next click the icon (below the arrow) to the right of the formula field for the X values. Once you do this the spreadsheet will be active and you can highlight cells A5 to A95. **Do not highlight column headings, just data points.** Then click the same icon next to the formula field to return to the data series window.

e. Do the same for the Y values (B5 to B95).

f. If you wish to move the graph after the data has been selected, right click the chart area and select “Move Chart” which will allow you to put it into a new sheet. This is recommended because it will allow you to place it out of the way of your active sheet and has a convenient chart tab at the bottom next to your sheet tabs.

g. Adding Axes: Select the layout tab and choose an option that allows the horizontal and vertical axes to be created and name them time (seconds) and velocity (m/s) respectively.

\*If Excel 2003 is used refer to the instructions at the end of the document.

1. **Modeling Position with Drag Force:**
2. To add a column to calculate position in meters, add a column label in cell G4: “Position (m)”. **Do not include quotation marks.**
3. Specify the initial position of the object in cell G5, which can be any positive or negative coordinate value. For now, enter “0” in G5.
4. The following cell will calculate final position by first calculating the displacement from the initial position at time = 0 seconds to the final position at the end of the first time interval, and then adding it to the previous position cell. This is equal to the average velocity multiplied by the time interval. Enter “=(B5+B6)/2\*$D$2+G5” in cell G6 and press enter. Click one time back on cell G6 and hover the cursor over fill handle in the lower right-hand corner of the cell and double click the fill handle.
5. In order to improve the precision of the model, more data points must be used. A factor of 10 decrease in the step size (time interval) will decrease the error by a factor of 100. Therefore, we will use 1000 calculating cells to track all variables. Jump to the bottom of the sheet by clicking on any cell in the computing array (rows 5 through 95). While holding down the “Control” key, press the down arrow to jump to the bottom of the array. Highlight the entire bottom row of data (A95:G95) and then rest the cursor over the fill handle. Drag the formulas down to row 1005.
6. Modifying the **Velocity vs. Time** graph: Right click the chart area of the graph and choose the option to “Select Data”. Choose Edit for the only existing series on the graph (velocity vs. time). The formulas should read: “='Sheet 1!’$A$5:$A$95” for the X values and “='Sheet 1!’$B$5:$B$95” for the Y values. Change the 95 to 1005 in the formula (or go back to the active sheet and replace the entire formula by highlighting to the bottom, preferably using keyboard short cuts “Control” + “Shift” + “toggle down arrow”.
7. Make a Position vs. Time graph: To do this simply copy and paste the Velocity vs. Time graph by right clicking the existing V-t graph, selecting copy, and then paste the graph into the active sheet by pressing “Control” + “v” or by right clicking the chart area and selecting paste. Then right click the pasted graph and choose “Select Data” and choose “Edit” the series containing the velocity vs. time data. Keep the X values the same but change the Y values to Position by simply changing each B to a G in the formula. If the formula gets corrupted, erase the entire Y values and highlight/select cells G5:G1005. The correct formula should read: “='Sheet 1'!$G$5:$G$1005”. Press OK until all dialog boxes are closed and the Position vs. Time Graph can now be edited to be labeled correctly.
8. **Modeling a Direction Change with Drag Force:**
9. In order for gravity and drag force to produce a direction change, the object must first be directed upward (in the negative direction in this case). Enter “**-**20” in cell B5 to represent an object thrown upward (in the negative direction) at 20 m/sec.
10. Now the drag force is still directed upward, which is incorrect, because it produces an acceleration less than 9.8 m/s2. This is because no mechanism exists to reverse the direction of the drag force when the direction of motion changes. To add this mechanism, an IF function must be used to tell the drag force to reverse directions when the velocity reverses directions. Since the drag force is subtracted in the Net Force Column (E), making the magnitude of the drag force (column D) negative will reverse its direction whenever the velocity is negative. To accomplish this, enter “=IF(B5<0,-$C$2\*$B$2\*B5^2,$C$2\*$B$2\*B5^2)” into cell D5 and press enter. This will make the drag force reverse directions when the velocity is negative.
11. Click back to cell D5 and then double click the fill handle in the lower right corner of the cell to send the corrected formula to the bottom of the page.
12. Note, by making either the drag coefficient or cross-sectional area values in row 2 equal to 0, the spreadsheet will produce ideal freefall graphs that can have any initial velocity or position specified in Row 5.
13. **How to automatically display the theoretical time:**
14. Choose a drop site and measure the height using a mass on a string. Heights higher than 3 meters produce longer drop times and therefore smaller amounts of error due to human timing, although slow motion video can also be used to measure drop time. Enter “Height (m)” in cell G1 and then enter the measured height in meters in cell G2 and enter “Theoretical Time (sec)” in cell H1.
15. Use a small time interval, so that model error introduced is minimal. An IF function will be used to check to see if the parachute has theoretically reached the ground. This can be done by typing: “=IF(G5>=$G$2,0,A5)” in cell H5, press enter, click back on that cell and double click the fill handle. This will display every time before the parachute hits and leave a 0 in the cell for all points after it hits the ground.
16. Enter “=MAX(H5:H1005)” in cell H2 to pull the maximum time out of column H, which corresponds with the moment the parachute has fallen the height indicated in G2.

Enter the data for the skydiver’s cross-sectional area and drag coefficient in cells B2 and C2 to find the terminal velocity of the skydiver. (items are listed in table 1.1 below)

1. **Only one object can be viewed at a time**. Enter the data for a single object from the table of objects, their masses, cross-sectional areas, drag coefficients and time increments which are to be entered in cells A2 thru D2 on the spreadsheet to determine the terminal velocity. Scroll down to see the velocity level off and when it is holding constant the object has reached terminal speed. Or you may simply observe the velocity in the bottom cell (B95). Observe the progression of the net force and acceleration columns as the object is speeding up over time.
2. Record the actual terminal velocity for each item in the table below. Then determine the distance that each item has had to fall to reach a rate of acceleration of less than 0.1 m/s2 and enter it in the table.

**Table 1.1**: Objects to be simulated falling from rest in air to reach terminal velocity.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Falling Item | Mass (kg) | CSA (m^2) | Drag Coefficient | Time Increment (sec) | Terminal Velocity (m/sec) | \*Distance to reach a = 0.1 m/s2 (m) |
| Skydiver | 95 | 0.6 | 0.55 | 0.5 |  |   |
| skydiver (head first dive) | 95 | 0.12 | 0.55 | 1 |  |   |
| skydiver (open parachute) | 95 | 25 | 0.5 | 0.1 |  |   |
| Feather | 0.001 | 0.002 | 0.55 | 0.02 |  |   |
| Javelin | 0.8 | 0.0004 | 0.45 | 1.5 |  |   |
| penny (sideways) | 0.003 | 0.0000019 | 0.5 | 1 |  |   |
| penny (facing down) | 0.003 | 0.0002834 | 0.55 | 0.1 |  |   |
| piece of paper | 0.005 | 0.0616 | 0.6 | 0.01 |  |   |
| Bullet | 0.013 | 0.0000785 | 0.45 | 0.5 |  |   |

\*This is the criteria for (essentially) reaching terminal velocity to see how far the object must fall to reach its terminal speed since the acceleration never mathematically hits 0.

**Development Questions:**

1. This spreadsheet assumes uniform acceleration over the specified time interval. For this to hold true, what must be true about the size of the time interval?
2. Why is it useful to be able to change the time increment at the top of the spreadsheet?
3. What do the dollar signs in each formula mean?
4. How is the net force on the object determined and how does it relate to acceleration?

**Analysis:**

1. If an object is traveling straight up in air, how does its acceleration compare to 9.8 m/sec2? If it was falling straight down, how does its acceleration compare to 9.8 m/sec2? Assume gravity and air resistance are the only forces acting on it.
2. **The returning bullet:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Item | Mass (kg) | CSA (m^2) | Drag Coefficient | Time Increment (sec) |
| Bullet | 0.013 | 0.0000785 | 0.45 | 0.03 |

* 1. How long will it take a bullet to return to ground level?
	2. How fast will it be falling?
	3. How high will it travel? *Hint, the minimum value in column G is the maximum height because up is the negative direction.*
	4. How close to terminal velocity is the bullet when it returns to ground level?
1. **Foul Ball:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Item | Mass (kg) | CSA (m^2) | Drag Coefficient | Time Increment (sec) |
| Baseball | 0.143 | 0.00417 | 0.5 | 0.02 |

* 1. A baseball player pops a foul ball straight up at 35 meters per second, and the catcher must get in position and make the catch with his glove 2.0 meters above the ground. The starting height of the ball was 1.0 meter. What is the maximum height that the ball reaches? *Hint: Up is negative, and the ground is at a height of 0 meters.*
	2. How much time will the catcher have to get in position to catch the ball?
	3. If air resistance was disregarded, what would be the answers to part A and B?
	4. Baseball on Mars: How high would the same ball go on Mars with air resistance (assuming a drag coefficient of 1% of that on Earth)? *Hint: Modify the formula in the weight column to replace 9.8 with 3.71 m/s2.*
	5. How high will the same ball go on the moon with no air resistance and g=1.62 m/s2? *Hint: make sure the time interval is big enough for it to return to ground level.*
1. **Meteorology Application: Hail stones:** Updrafts in the upper atmosphere are capable of lifting rain drops that cycle through masses of water vapor and accumulate mass layer by layer until they fall from the sky as hail. The same factors that produce hail are risk factors for tornados.
	1. The maximum theoretical size of the hail depends directly on the velocity of the updrafts. How fast must the updrafts be to produce spherical hail stones to reach the National Weather Service threshold of “damaging hail” (which is stones equal to or greater than 2.5 centimeters (on inch) in diameter)?  *The drag coefficient can start at 0.5 for a sphere and the density of ice is 920 kg/m3. To solve enter “Diameter (m)” in cell E1 and enter 0.025 in cell E2. Use a formula linked to this to calculate the mass and cross-sectional area of a sphere and observe the terminal velocity. Mass is density times volume: Type “=920\*4/3\*pi()\*(E2/2)^3” in cell A2. Then type “=pi()\*(E2/2)^2” for the Cross-sectional area in cell B2.*
	2. Very large hailstones typically form when conditions are such that multiple smaller spherical stones stick together forming a larger mass of irregular shape. This bumpiness can increase the drag coefficient and increase the cross-sectional area for a given quantity of mass. This effectively lowers terminal velocity while the larger mass ensures greater damage to objects impacted by such stones. Hypothesize, sketch and simulate the terminal velocity of such stones and research images of large hailstones and draw some conclusions after simulating and observing them falling on the spreadsheet model. You will have to experiment with the numbers to try to fit your sketch so use the data from part a as a reference.
2. **Biology Research Application:** Research the size and mass of the following tree dwelling animals compared to a ground dwelling counterpart of similar size to simulate its terminal velocity to draw conclusions about the evolution of structure and function of overall body type for tree dwelling vs. ground dwelling animals.
	1. **Flying squirrel** versus a **sewer rat** of comparable body size. **Essential question:**  Discuss in writing and produce data to support the claim that the structure and function of body adaptations the flying squirrel versus sewer rat evolved as a means of providing a survival advantage in their living environment.
	2. **Healthy cat** versus a **significantly overweight cat (by 50 percent of body weight)** of comparable cross-sectional area. **Essential question:**  Discuss in writing and produce data to support the claim that a cat will survive a fall from atop a tall building provided they
	3. **A common tree snake** versus a **puff adder** of comparable mass and cross-sectional area. **Essential question:**  Discuss in writing and produce data to support the claim that the adaptations that are advantageous in one environment may be detrimental in another. Puff adders tend to inflate their body size and girth to scare off predators. Tree snakes tend to be long and skinny to navigate branches (with the secondary benefit of producing higher drag coefficients).

**Hands-on Design Competition Spreadsheet Modeling Application:**

**Parachute Drop Building Competition:**

Use an internet search to identify web sites with instructions for building parachutes with simple household materials, such as paper, tape, string and a small mass such as 3 to 8 pennies as a payload. Approximate its effective cross-sectional area while it is falling by holding the strings and pulling it in air so that the parachute inflates. Measure the mass of the entire parachute-payload system with a balance to maximum precision. Use an internet search to try to estimate the drag coefficient. This can be tricky, as different web sites use different formulas. Keep in mind that our drag coefficient is drag force divided by (mass\*velocity^2). Use the spreadsheet model with your data to predict the theoretical time for the fall. Then drop the parachutes while carefully measuring the time. Compare the experimental and predicted times to see who got the closest. For an alternative contest, each group tries to build a parachute that will have the longest experimental fall time.

(be sure to use SI units for all measured quantities to be entered)

**Teacher Produced Questions: (Developed as part of module training)**

1. Paste each question received by member teachers here once validated.