Angular Motion in Cars:

**Objective:** To calculate the speed of a car given the gear ratios and RPM’s, and to determine the time necessary to get the car up to speed in various situations.

**Discussion:** There is a significant amount of basic physics at work in an automobile engine. Cars have engines that burn gasoline inside pistons (internal combustion engines). There are several steps involved in extracting the energy from gasoline. The gasoline gets injected into the piston (an enclosed cylindrical container in the engine with a movable arm) and compressed. It is then ignited with a spark from the spark plug, at which point it expands as it ignites and increases in temperature and pressure. This is called the power stroke\*. After the power stroke, the piston closes again and exhausts the combustion products and begins the cycle again. The more quickly this happens, the faster a car’s engine will cycle. Because this process is limited in how quickly the movable arm can move back and forth, cars would have a very limited range of velocities. However, cars have a transmission that allows the engine to turn the tires more quickly (in higher gears) while the engine moves at slower speeds. If you have ever ridden a bike that can shift gears you know how different speeds and conditions call for different gears.

Cars have a tachometer that measures engine speed in RPM’s. If the transmission is manual, the driver decides when to shift gears based on the RPM’s and need for torque. If it is automatic, an onboard computer shifts based on driving patterns, fuel flow rate and RPM’s. If the engine speed is known as well as the gear ratio, the rotational rate of the tires can be found. If this is known as well as the radius of the tires, then the speed of the tires and thus the speed of the car can be determined. The overall ratio for each gear must be multiplied by the drive ratio of the car, which in this case is 4:1.

\*Note that power is only delivered to the car when it is completing the power stroke, which is one quarter of the time. Therefore most engines have at least four cylinders in order to ensure that one is always on its power stroke.

**Governing Equations:** All variables are in SI base units.

1.1 

1.2  ** = angular velocity

1.3  V = linear velocity, r = radius

1.4  T = torque, F = force, r = radius

1.5  FNet = net force, m = mass, a = acceleration

1.6  t = time, Vf = final velocity, VI = initial velocity, a = acceleration

1.7  F = force, m = mass, g =9.8 m/sec2, =angle incline

1.8  P = Power, W = work, t = time

**Setting Up:** Open Microsoft Excel®

1. Set up your spreadsheet exactly as you see below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | A | B | C | D |
| 1 | Gear | Gear Ratio | Peak Transmission Torque (Nm) | Transmission Torque at Red Line (Nm) |
| 2 | 1 | 3.828 | 1500 | 829 |
| 3 | 2 | 2.36 | 925 | 511 |
| 4 | 3 | 1.685 | 661 | 365 |
| 5 | 4 | 1.312 | 514 | 284 |
| 6 | 5 | 1 | 392 | 217 |
| 7 | 6 | 0.793 | 311 | 172 |
| 8 | R | 3.27 | 1286 | 710 |

1. Add the following cells to your spreadsheet in cells C11 and 12 and D11 and 12:

|  |  |
| --- | --- |
| Tire Diameter (inches) | Car Mass (kg) |
| 30 | 500 |

1. Add the following columns in the cells indicated below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E |
| 13 | Gear |  |  |  |  |
| 14 | Engine RPM's | Engine ω (rad/sec) | Tire ω (rad/sec) | Tire speed (m/sec) | Car Speed (mph) |

Cell B13 will be the gear shifting cell. Gears 1-6 can be chosen and the car speed will be calculated for the various engine RPM’s. If reverse is desired, the formula to be written will be such that (-1) can be entered as the gear for reverse.

1. Engine RPM’s can range anywhere from 600 RPM’s to 7500. This may seem like a wide range, but if there were no gears, the cars maximum speed would be 35 mph. Additionally, the car would only reach 35 mph while at the maximum attainable engine speed. To enter the possible engine speeds for the car, start with 600 in cell A15 and 610 in cell A16. Highlight both cells and drag the fill handle down until it reads 7500 (at cell 705).
2. The engine speed must be converted to radians per second in order to be converted to linear speeds using Equation 1.1. In cell B15 divide rotations per minute by 60 and multiply 2 times pi (3.14159) by entering “=A15/60\*3.14159\*2”. Pi can also be entered into the formula as “Pi()” if desired. Double click the fill handle.
3. Now the tire speed must be calculated. This depends on several things: The drive ratio of the transmission (the engine rotates 4 times for every one time the transmission rotates in this case), the gear ratio of the current gear engaged in the transmission, and the angular velocity of the engine. This is all given information that can be used with Equation 1.2, but should be referenced in such a way that the spreadsheet will select the correct gear ratio depending on the value entered in cell B13. In order to do this several imbedded IF functions will be used to account for the seven possible gears (1-6 and reverse). Enter the following formula into cell C15: “=B15/4/IF($B$13=1,$B$2,IF($B$13=2,$B$3,IF($B$13=3,$B$4,IF($B$13=4,$B$5,IF($B$13=5,$B$6,IF($B$13=6,$B$7,IF($B$13=-1,-$B$8)))))))” and double click the fill handle.
4. The rotational velocity can relate to the linear velocity and the radius of the tires using Equation 1.3. In cell D15 enter “=C15\*($C$12/12\*0.305/2)” to convert the tire diameter to radius in meters and then multiply by the angular velocity in cell C15.
5. Once the velocity of the tires is calculated it can be converted to miles per hour in column E. In cell E15 enter “=D15\*3600/1000/5\*3.107”. Double click the fill handle of all remaining cells to carry the formula down to 705.

The velocity profile of the car at any gear can be calculated. Shift the gears by changing the value in cell B13 to see the different velocities the car can reach for a given engine speed and gear. The time that the car will take to reach the speeds indicated depends on the torque and power output of the engine. The torque is given at the optimum range (approximately 5000 RPM’s) and at the redline (7200 RPM’s).

If a car with a traditional (non-continuous) transmission is to maximize acceleration it needs to be driven in the gear with the highest torque, or the lowest gear, for the longest amount of time possible. This means that the gear should not be shifted until the redline RPM’s are reached. The torque delivered to the tires is going to relate to the force that is pushing the car forward. Since torque varies with engine speed, it cannot be assumed that the optimum torque will always be present. Therefore, the optimum torque will be averaged with the redline torque and this value will be used.

1. Set up the following column headings in the cells indicated:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | E | F | G | H |
| 1 | Force (N) | Redline Force (N) | Acceleration (m/sec2) | Redline Acceleration (m/sec2) |

|  |  |  |
| --- | --- | --- |
|  | I | J |
| 1 | Average Acceleration (m/sec2) | Time (sec) |

1. In cell E2 enter the following formula: “=C2/($C$12/2/12\*0.305)”. This will calculate the force resulting from the torque provided at the optimum range using Equation 1.4.
2. In cell F2 the same can be done for the redline torque by entering the following formula: “=D2/($C$12/2/12\*0.305)”. Drag and drop the fill handle to complete the calculation for each gear.
3. The optimum and redline accelerations can be determined using Newton’s Second Law, or Equation 1.5. Enter “=E2/$D$12” into cell G2 and “=F2/$D$12” into cell H2. Drag and drop the fill handle to complete the calculation for each gear.
4. The acceleration to be used will be the average of the two accelerations just calculated. Enter “=AVERAGE(G2:H2)” into cell I2. Double click the fill handle.
5. The time to go from one gear to the next depends on the speed of the car at the time the shift is made. If we assume that it takes no time to complete a shift and that the shift is made at the redline (7200 RPM) then we can predict the time elapsed during each gear. This will be calculated in column J. If the car starts in first gear and is cycling at 600 RPM, the time can be calculated to reach the redline RPM’s using Equation 1.6 with the following formula in cell J2: “=IF($B$13=1,($D$675-D15)/I2,0)”.
6. If the shift is made at the redline, then the car speed in cell E675 in first gear should not change when the shift is made to second gear. Shift to second and find the same car speed in column E. This cell will be the initial velocity in cell J3 and can be found in cell E399. Enter “=IF($B$13=2,($D$675-D399)/I3,0)” into cell J3. This is the time the car spends in second gear. Do the same for subsequent gears and answer the questions below. The formulas for each cell should be as follows: J4: “=IF(B13=3,(D675-D469)/I4,0)”;

J5: “=IF(B13=4,(D675-D515)/I5,0)”; J6: “=IF(B13=5,(D675-D503)/I6,0)”

J7: “=IF(B13=6,(D675-D526)/I7,0)”; J8: “=IF(B13=-1,(D675-D15)/I8,0)”.

**Development Questions:**

1. Explain how an IF function can help you change gears.
2. Why are the accelerations from the peak and redline torque averaged rather than using the peak transmission torque by itself?
3. How many cells are actively performing calculations on this spreadsheet when a single data value is changed?

**Analysis:**

1. What is the top speed of the car if the engine cannot exceed 7200 RPM?
2. At which gear does the car accelerate the fastest and why? Also indicate the time it takes to reach 7200 RPM's.
3. What happens to the time it takes the car to reach 7200 RPM in first gear if larger radius wheels or tires are put on the car? What is the reduction in force at the tire in first gear at peak transmission torque if tire radius is increased by 8 inches?
4. Which gear has the highest speed? How does the torque of this gear compare to that of the other gears?
5. How much time will it take the car starting from rest with 30 inch diameter tires to reach its top speed in sixth gear (assume no air resistance)? *Hint: Add up the times for each gear up to 6th.*
6. In which gear does it reach half its top speed? How much time will it take to reach ½ of its top speed?
7. Explain why the car takes less time to go from 0 to ½ its top speed than it does to go from ½ its top speed to its top speed. How would air resistance affect getting to the top speed?
8. What is the kinetic energy of the car at ½ its top speed? At its top speed?
9. Calculate the power of the car in watts and horsepower to get to half its top speed and then to get to its top speed. (746 watts = 1 hp)
10. Is it realistic if in a high speed chase in an action movie the driver traveled in reverse during a high speed chase passing cars on the highway? What is the top speed in reverse if the engine cannot exceed 7200 RPM?

**Challenge Questions:**

1. How fast will a car accelerate up an incline of 10.0 degrees at its optimum torque? The car must lift its weight up the incline. You can ignore the force of friction on the car. Use equation 1.7.Use Newton’s 2nd Law and draw a free body diagram of the car on the incline.
2. What would the engine RPM’s become if the driver was traveling 50 miles per hour and accidentally downshifted to first gear? Use goal seek. *It is found under the "Data" tab in "What if Analysis" (or the tools menu for 2003 or earlier)*.
3. Engine Braking: If the accelerator is not pressed, only a minimal amount of fuel is injected into the engine. If the engine is cycling quickly and limited fuel is being injected. Explain at least 3 energy transformations (and the initial and final types of energy for each) that are taking place in slowing the car.